Modeling Major Flank Wear in Turning;
The Mechanistic Cumulative Wear-Energy Model

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Introduction

The goal of this work is to present a mechanistic model describing the cumulative energy needed to perform a particular cut as a function of cut path distance and cutting parameters. This cumulative energy model will be developed by first deriving a cutting force model that is respective of the tool's changing wear state. Throughout the derivation, assumptions and their validities will be presented and discussed. After the final cumulative energy model is defined, it will be evaluated for correlation to the major flank wear of the cutting tool at multiple distances throughout the tool wear test. It is anticipated that the relationship between the cumulative cut path wear-energy and the major flank wear is related by a scalar, k. To make this evaluation, cutting data from twelve turning insert tests is analyzed and compared to the model outputs as a function of cut distance. This model is offered as an alternative to the Taylor tool life equation. Unlike the Taylor equation which produces a single time value for tool life, this model can produce a theoretical wear-energy curve with the features of break-in, linear stable wear, and accelerated wear modes. This model is also capable of determining the allowable cut distance based on the allowable major flank wear.

Mathematical Theory and Development of a Mechanistic Cutting Force Model

When developing a theoretical model for an engineering system, it is necessary to begin with conservation relationships or geometric limitations. The cutting tool system presents both of these items and, consequently, the development of constraints is rather elementary. For the development of this model, friction will not be considered. This is a valid assumption for the intent of this model since the cutting tool material is assumed to have no propensity for adhesion to the workpiece (different material types, i.e. the tool is ceramic). In the absence of adhesion changing as a function of wear state, the effect of kinetic friction is only a scalar value that is useless from the perspective of correlation to a developing wear curve. The intent of this model is to generate a relative cut energy curve that reflects the current wear state behavior. If the model were intended to identify the true cumulative cut path energy, the frictional effects between the tool facets and chip could be considered. Figure 1 is an illustration of the cutting tool-workpiece interface and identifies variables that will be used to develop a cutting force model from the system.

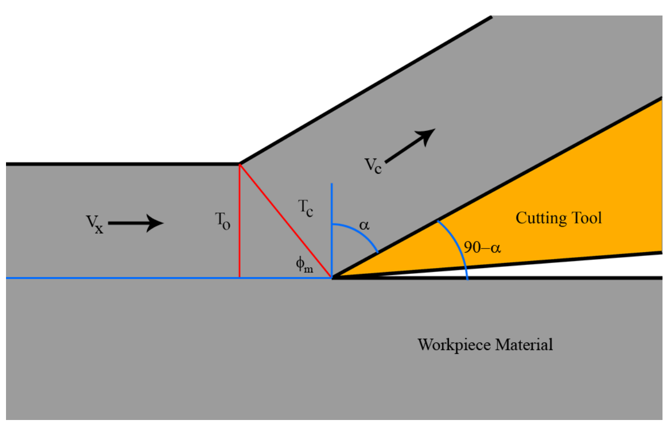

Figure 1. Tool Illustration Identifying Geometric Variables

Chip Ratio as an Autonomous Function of Surface Feed Rate

The first relationship in this system that requires identification is the chip ratio. This variable represents the relative thickness of the depth engagement to that of the departing chip thickness. This ratio is crucial to understanding the geometry of the maximum shear plane (discussed later). The chip ratio is found as:

$$R = \frac{T_o}{T_c} \tag{1}$$

Where T_o is the depth of tool engagement and T_c is the chip thickness.

After recognizing the chip ratio, it can be stated that, as a result of volume conservation, that the mass flow rate of the chip is equal to the mass flow of the incoming material. Specifically, the system can be equated to an incompressible control volume whereas the mass flow of incoming material is equal to the mass flow exiting:

$$\dot{m}_i = \dot{m}_o \tag{2}$$

Recognizing that the unit width of the material cancels from the mass flow relationship, the conservation equation becomes:

$$T_o V_x = T_c V_c \tag{3}$$

Where V_x and V_y are the x and y components of chip velocity. It can be noted that the x component of velocity is the surface feed rate of the workpiece. A geometric constraint can now be applied to define V_y in terms of V_x:

$$V_y = V_x \tan(90 - \alpha) \tag{4}$$

Where α is defined as the orientation angle (in degrees) of the tool with respect to the x-axis. The mean velocity of the chip can then be given as:

$$V_c = \sqrt{V_x^{\,2} + V_y^{\,2}} \tag{5}$$

After applying the conclusion of volume constancy given by equation 3, equation 5 can be rewritten as:

$$T_o V_x = T_c \sqrt{V_x^{\,2} + V_y^{\,2}} \tag{6}$$

Further simplifying equation 6 to exist in terms of V_x:

$$T_o V_x = T_c \sqrt{V_x^{\ 2} + (V_x \tan(90 - \alpha))^2} \tag{7}$$

Finally, recalling the definition for chip ratio from equation 1, R is put in terms of V_x:

$$\frac{T_o}{T_c} = \frac{\sqrt{V_x^{\ 2} + (V_x \tan(90 - \alpha))^2}}{V_x} \tag{8}$$

In this way, the chip ratio is an autonomous function of the surface feed rate, V_x.

Maximum Shear Plane Angle as an Autonomous Function of Surface Feed Rate

Like the chip ratio, the maximum value for shear plane angle, ϕ_m, exists as a function of surface feed rate. However, unlike the chip ratio, the maximum shear plane angle only defines a limit such that $0 < \phi \le \phi_m$. From the geometry depicted in Figure 1, the maximum shear plane angle can be defined as:

$$\phi_m = 180 - 90 - \sin^{-1}\left(\frac{T_o}{T_c}\right) \tag{9}$$

$$\phi_m = 90 - \sin^{-1}\left(\frac{T_o}{T_c}\right) \tag{10}$$

Substituting for the chip ratio this statement becomes:

$$\phi_m = 90 - \sin^{-1}(R) \tag{11}$$

$$\phi_m = \cos^{-1}(R) \tag{12}$$

Recalling equation 8, this can be put in terms of the surface feed rate, V_x:

$$\phi_m = \cos^{-1}\left(\frac{\sqrt{V_x^{\ 2} + (V_x \tan(90 - \alpha))^2}}{V_x}\right) \tag{13}$$

Complex Converse to the Maximum Shear Plane Angle

As ϕ_m approaches vertical, its complex converse, ϕ_i, measured from the vertical orientation with respect to ϕ_m approaches 90º according to equation 14:

$$\phi_i = \cos^{-1}\left(\frac{V_x}{\sqrt{V_x^{\ 2} + (V_x \tan(90 - \alpha))^2}}\right) \qquad (14)$$

Figure 2 illustrates ϕ_i with respect to the vertical and ϕ_m.

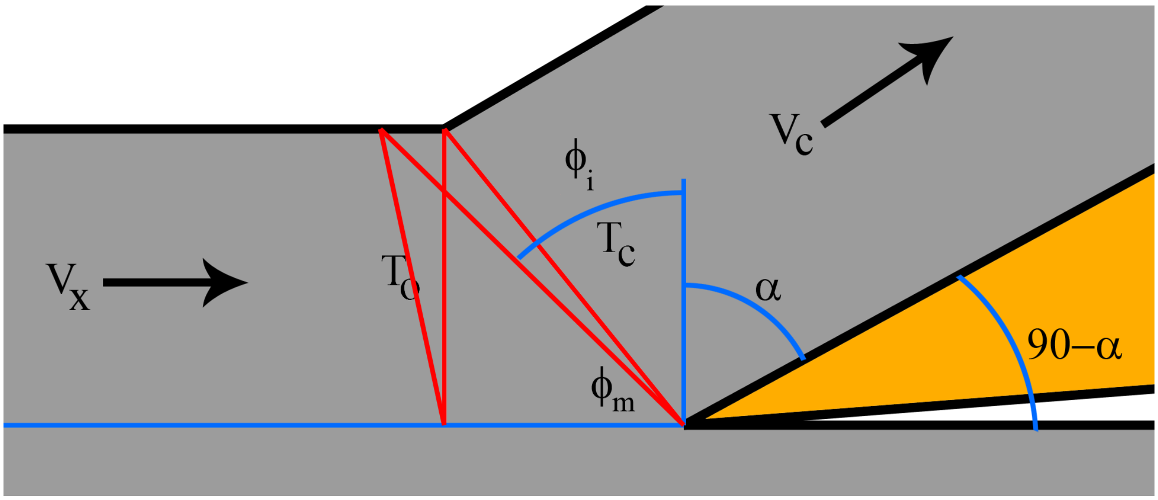

Figure 1. Illustration of the Complex Converse Angle, ϕ_i

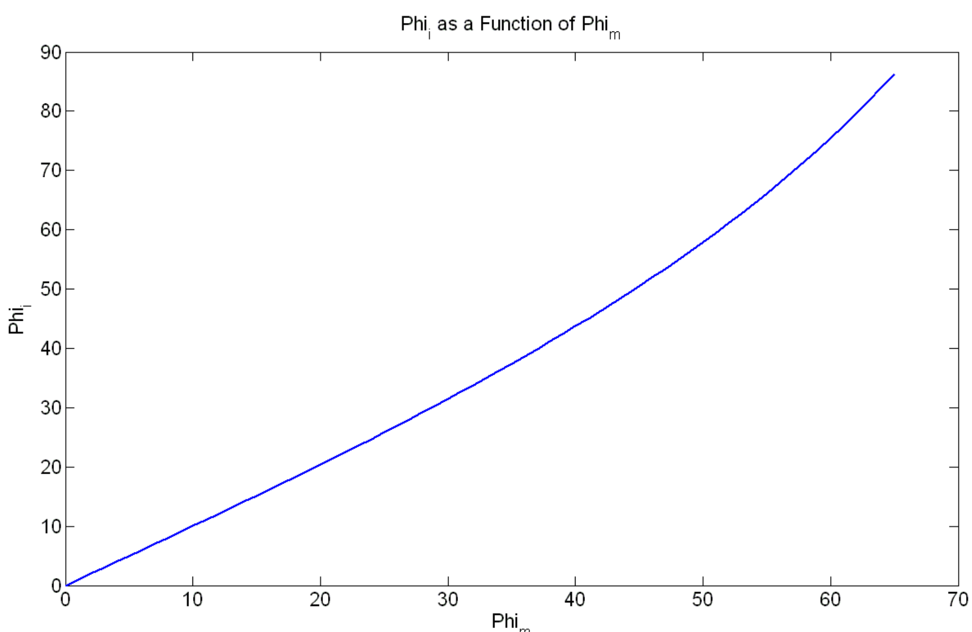

Figure 2. ϕ_i as a Function of ϕ_m (angles in degrees)

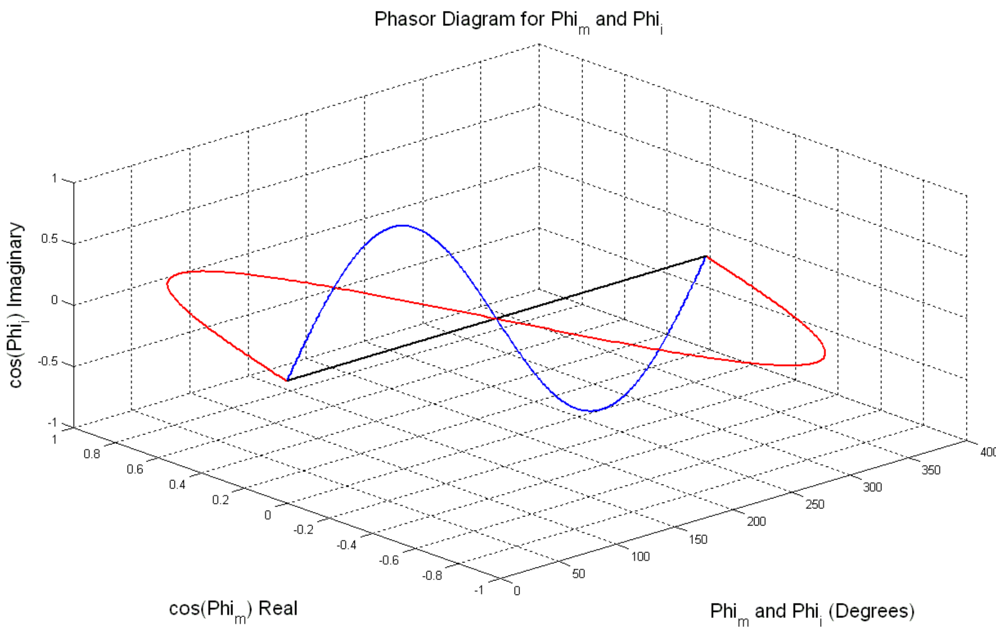

Figure 3. Phasor Diagram for Complex Phi

As can be seen, the real and imaginary components of the complex ϕ are in phase. The actual shear angle is assumed to be defined by ϕ_i since the plane defined by ϕ_i changes as a function of a wearing tool and an increasing ϕ_m. This is beneficial from the perspective of developing a model which incorporates change in shear plane as a function of cut path distance (cumulative wear).

Calculating a Theoretical Maximum Cut Force: Incorporating Shear Yield Strength

After the identification of the maximum shear plane angle, it is possible to find the area of the corresponding maximum shear plane. It can be noted that the maximum shear plane is orthogonal to the plane of the tool face. This can be done by considering the shear relationship:

$$\tau = \frac{F}{A_m} \tag{15}$$

Where F is the applied force and A_m is given in terms of the surface feed rate as:

$$A_m = T_c W \tag{16}$$

and $T_c = \dfrac{T_o}{\cos(\phi_i)}$.

W is the width of the tool contact area (engagement width). For the case of non rectangular tools, A_m must be calculated by identifying the engaged contact width with the workpiece and integrating a parametric function, $f_t(t)$, defining tool curvature over W:

$$A_m = T_c \int_0^W f_t'(t)dt \tag{17}$$

This relationship is shown in a parametric form for the sake of generality. Specifically, if $f_t(t)$ is a helical function, such as an endmill, the Euclidean counterpart to $f_t(t)$ would take the form $f_t(x, y, z)$ and would require a triple integral to be evaluated in three space.

Assuming that the yield strength of the workpiece is less than that of the tool, this value can be applied to find the force by rearranging equation 14 and substituting τ_Y for τ :

$$F = \tau_Y A_m \tag{18}$$

After making this substitution, the area of the maximum possible shear plane, A_m, can be defined by the chip thickness, T_c and the tool engagement width, W.

$$F = \tau_Y T_c W \tag{19}$$

Recall, however, that the chip thickness, T_c, is a function of the surface feed rate, V_x and the depth engagement, T_o:

$$F = \tau_Y W\left(\frac{T_o}{\cos(\phi_i)}\right) = T_o \tau_Y W V_x^{-1} \sqrt{V_x^2 + (V_x \tan(90 - \alpha))^2} \tag{20}$$

Where $\phi_i = \cos^{-1}\left(\frac{V_x}{\sqrt{V_x^2 + (V_x \tan(90 - \alpha))^2}}\right)$ (from equation 14)

As with friction, the addition of flow stress to this model is unnecessary. This is because the model is intended to produce a trend of relative cumulative wear-energy. In this sense, the addition of flow stress to the model adds complexity but only produces a scalar offset (since the flow stress only changes as a function of cutting rate). Therefore, shear yield will be used as a relative stress for this model.

Extension of the Force Model to a Cumulative Cut Path Energy Model

Integrating this force over the tool path, s, the total energy of the cut can be determined as so:

$$E = \int_0^d F \cdot ds \tag{21}$$

Where E is defined as path energy. Since wear takes energy to occur, the path energy, E, is directly related to the wear state at any point, d, along the cut path. However, the force, F, is also a function of the tool's wear state. This complicates the relationship such that wear and E are interdependent variables. As defined, force is a function of depth and width engagements, the surface feed rate, tool orientation (α), and the shear yield strength of the workpiece material.

Dynamic Wear Effects and the Incorporation of a 1st Order Wear Assumption

When the tool wears, the tool orientation at the workpiece-tool interface is changing. Therefore, it is evident that the variable linking wear and path energy is the tool orientation angle, α. At this juncture, the orientation angle, α, has been further constrained by dependence on cut distance, d. Assuming that the tool wears linearly in a stable wear mode, the change in α can be defined as a 1st order function of d and V_x:

$$\alpha_d = Cd \cdot V_x + \alpha_o \tag{22}$$

Where α_o is the initial (unworn) orientation angle, and C is a constant defined for the given tool and workpiece materials. C has units of m^2/s and $d \cdot V_x$ represents normalized surface area removed over time (i.e. independent of engagements).

After this relationship is declared, α_d is substituted into equation 18 to redefine the force as a function of cut path distance:

$$F = T_o \tau_Y W V_x^{-1} \sqrt{V_x^2 + (V_x \tan(90 - \alpha_d))^2} \tag{23}$$

Recalling that the cut path energy is the integral of force times the current cut path distance, the current cut path energy can then be calculated as:

$$E_d = d \cdot T_o \tau_Y W V_x^{-1} \sqrt{V_x^2 + (V_x \tan(90 - \alpha_d))^2} \tag{24}$$

This model can also be redefined as the velocity changes by adding the periods of constant velocity and engagement (i.e. $E_d = E_{d1}(V_{x1}, T_{o1}, W_1) + \ldots + E_{dn}(V_{xn}, T_{on}, W_n)$) in this sense, if the window over which velocity is approximated to a constant value is sufficiently small, the approximate cumulative energy will be reflective of the current tool state. However, if velocity is constant during the entire process (such as the case with this turning lab), the value of energy exists as a nongregarious function of the linear cut distance, d, and the cutting conditions.

Tool failure would be defined by the measured wear level breeching a threshold correlation coefficient between the current energy level and wear magnitude such that the two lines (energy and wear magnitude) begin to take on a higher order relationship or a relationship that must be defined by a transcendental approximation such as the exponential function. Wear magnitude must be a discrete set which adheres to the same sampling interval as the calculation of current energy. This sampling rate must be determined as a function of distance and for the method to work, the step size must be sufficiently small to approximate velocity or engagement changes to constant values. Fortunately for this turning lab, this is not a consideration.

Experimental Parameters

There were three turning experiments performed in order to verify this model. These tests were run on three separate days by different groups of individuals. The parameters for the Monday and Friday tests were identical and the parameters for the Wednesday test were different. The tool type was Tungsten Carbide and the workpiece material was AISI 4140 steel. Tables 2 and 3 contain the test cutting conditions. Table 1 contains the English standard unit system and Table 2 contains the SI unit conversions. For this discussion, the SI unit conversions will be utilized.

Lab Test Day	Surface Feed Rate (ft/min)	Axial Feed (in/rev)	Cross Feed (in/rev)	Retract (in/rev)
Monday	900	0.014	0.014	0.014
Wednesday	1100	0.014	0.014	0.014
Friday	900	0.01	0.01	0.01

Table 1. Test Feed and Speed Parameters (English Units)

Lab Test Day	Surface Feed Rate (m/s)	Axial Feed (m/rev)	Cross Feed (m/rev)	Retract (m/rev)
Monday	4.572	3.556e-4	3.556e-4	3.556e-4
Wednesday	5.588	3.556e-4	3.556e-4	3.556e-4
Friday	4.572	2.54e-4	2.54e-4	2.54e-4

Table 2. Test Feed and Speed Parameters (SI Units)

In order to identify the total cut distances, the z-axis stroke of the lathe for each pass is divided by the stroke by the per revolution (Axial Feed) and multiplied by the current circumference. Specifically, this is calculated as:

$$d = \sum_{i=1}^{n} \pi D_i \left(\frac{S_i}{F_a}\right) \tag{25}$$

Where D_i is the current pass diameter, S_i is the axial stroke of the current pass, and F_a is the axial feed rate for the test. This relationship was used to determine the cumulative distances in table 3:

	Cut Path Distance (m)
Monday	1030.833384
Wednesday	565.8828811
Friday	1764.54276

Table 3. Calculation of Cumulative Cut Distance from the Axial Stroke and Pass Radii

Cumulative Cut Path Energy Model Evaluation

Since the application of this mechanistic model was done on a simple turning system without a large change in inertial mass, the dynamics of the machine system are assumed to be constant. Therefore, the cumulative energy is a linear function of distance and wear. This implies that the system does not have to be windowed and treated as a piece-wise cumulative function of discrete quasi-static states. The parameter α_0 is already known as the unworn orientation angle (Figure 1). The unworn orientation angle for this test is approximately 45°. The chip thickness, T_o, is defined as the depth engagement of the insert. Likewise, the contact width, W, is defined as the axial feed per revolution. The surface feed rate, V_x, is constant since the machine compensates the radial speed as a function of current radius. Finally, the shear yield strength of AISI 4140, τ_Y, is known to be 217.5e6 Pa. Although these values have been defined by the cut parameters, the constant, C, introduced in equation 21 must be identified. This constant is a function of workpiece and tool material types. For this work, the value of C is based on a Tungsten Carbide insert cutting AISI 4140 steel.

The outcome of the cumulative energy model is expected to follow the known wear behavior for cutting tools. This known behavior consists of periods of tool break-in, steady linearly increasing wear, and accelerated wear towards catastrophic failure. Catastrophic failure is defined as the fracture or melt-down deterioration of the tool. This theoretical wear curve is presented in Figure 2:

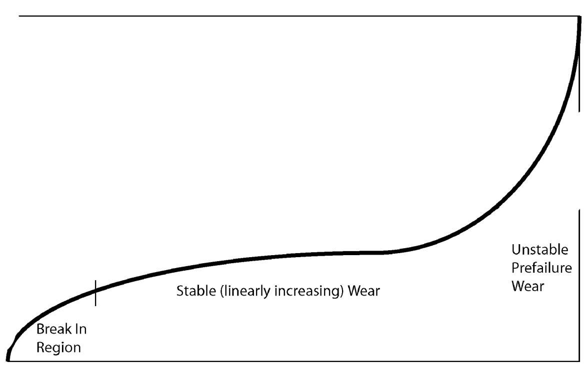

Figure 4. Theoretical Cutting Tool Wear Curve

The constant C is determined by performing an iterative fit in a least squares sense of the model defined in equation 23 to the lab test data. This value is found to be approximately 0.03. The parameters used to evaluate the model for each test are in Table 4 as follows:

	Monday	Wednesday	Friday
T_o (m/rev)	3.556e-4	3.556e-4	2.54e-4
τ_Y (Pa)	217.5e6	217.5e6	217.5e6
V_x (m/s)	4.572	5.588	4.572
W (m/rev)	3.556e-4	3.556e-4	2.54e-4
α_o ($^\circ$)	15	15	15

Table 4. Model Parameter definitions for Monday, Wednesday, and Friday Tests

In this way, C was defined for the tool and workpiece material combination. This fit was also performed on the data from Wednesday and Friday and produced similar values for C ranging from approximately 0.025 to 0.035. Since these values had a mean of approximately 0.3, this value was used for C when evaluating the theoretical model. The model was evaluated until a vertical asymptote was encountered. This asymptote occurs at an inflection (axis intersection) of the linear function of α_d shown in equation 23. It should be noted that the iterative solutions for C were based on a limited quantity of data collected from the three lab sections. In application, C could be recalculated throughout the test since material properties of both the tool and workpiece are dynamic (due to temperature, increasing surface work hardening, etc.). However, the resulting outcome of E_d (equation 24) is plotted as a function of d in Figure 3 for the test parameters of Monday, Wednesday, and Friday tests. For instance, the model could be manipulated to determine a planned total cut distance as a function of the desired major flank wear. Specifically, the model could be rearranged to identify the allowable cut distance for a major flank wear level of n meters:

$$kn_d = d \cdot T_o \tau_Y W V_x^{-1} \sqrt{V_x^2 + (V_x \tan(90 - \alpha_d))^2} \qquad (26)$$

For the case of AISI 4140 steel and a Tungsten Carbide insert, the value for the constant, k, is approximately 4e8 when the major flank wear, n, is measured in meters. Recall, that the definition of α_d is given according to equation 21.

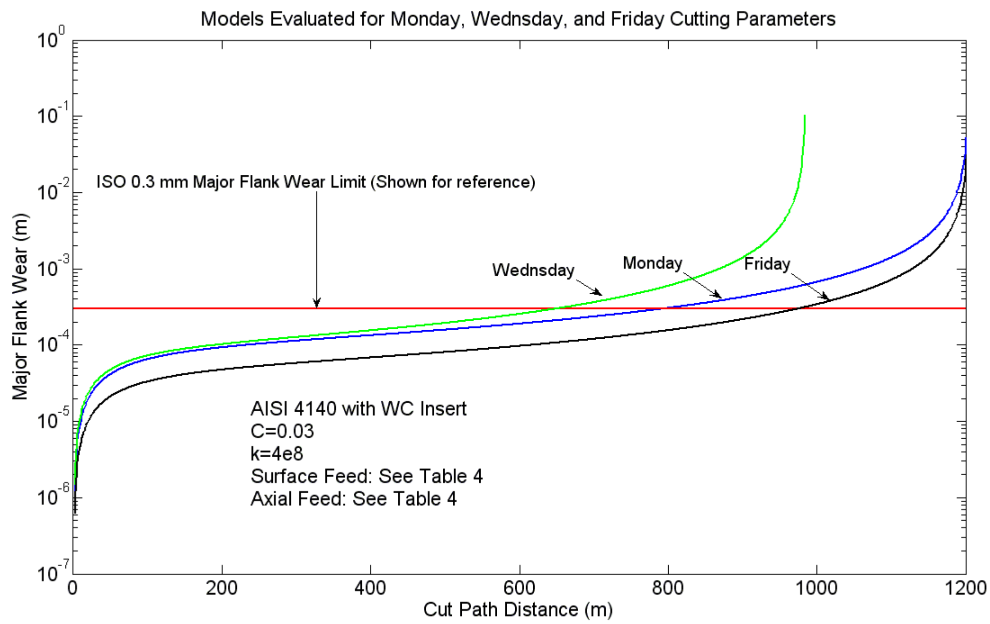

Figure 5. Cumulative Energy Models for Monday, Wednesday, and Friday Parameters

As expected, the model identifies energy curves reflective of the theoretical wear experienced by the tools. Recalling that the theoretical wear curve, shown in Figure 2, is an indication of the tool's state (break-in, linear wear, accelerated wear), the cutting energy requirement is directly related to the condition of the tool as a function of cutting force. However, it is interesting to note that the conditions of Friday's tests produce a lower expected life than those of Monday and Wednesday. This is curious since the cutting parameters of both Monday and Wednesday were more aggressive than those of the Friday test. This is attributed to the limited resolution available from this lab data. In a true wear test, the constant, C, could be recalculated and produce a new model wear curve projected from the current tool and workpiece condition. This feature would be particularly beneficial for use with composite material workpieces.

Energy Model Correlation to Major Flank Wear

The cumulative cut energy is a direct function of the wear state of the tool. Therefore, there exists a correlation between the behavior of the cumulative energy level and the current wear state of the tool. To investigate this correlation, the developed mechanistic model for cumulative cut energy will be contrasted against the curve of major flank wear on the cutting insert. The cutting insert used for these experiments is made from Tungsten Carbide. The ISO standard for tool failure is defined as 0.3 mm of major flank wear. Therefore, the tools are only run until this wear level is reached. The corresponding cumulative cut path energy curves are plotted over the same distance scale to show the strong correlation.

Figures 6 through 8 show plots comparing the experimentally measured major flank wear and the cumulative energy model evaluated at the same cut distance points. As is shown, there is a strong correlation between the state of major flank wear and the cumulative cut energy as a function of cut path distance.

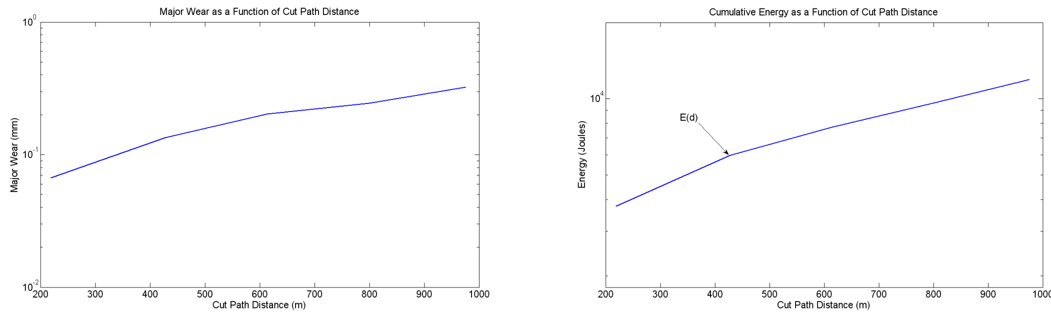

Figure 6. Experimental Major Wear Data (left) and Cumulative Energy Model (right) for Monday

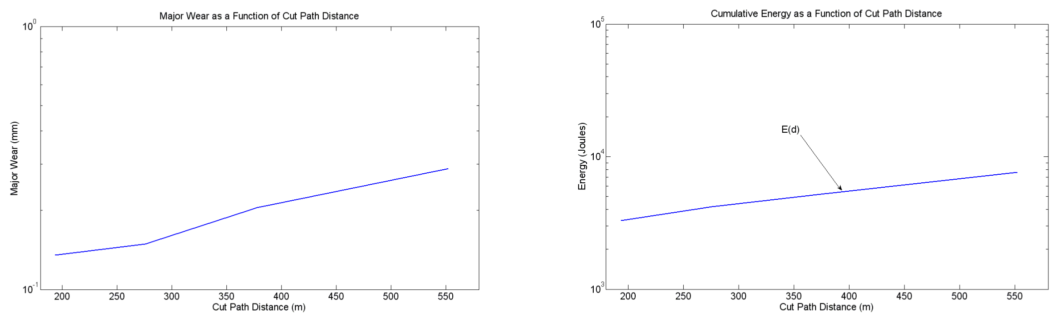

Figure 7. Experimental Major Wear Data (left) and Cumulative Energy Model (right) for Wednesday

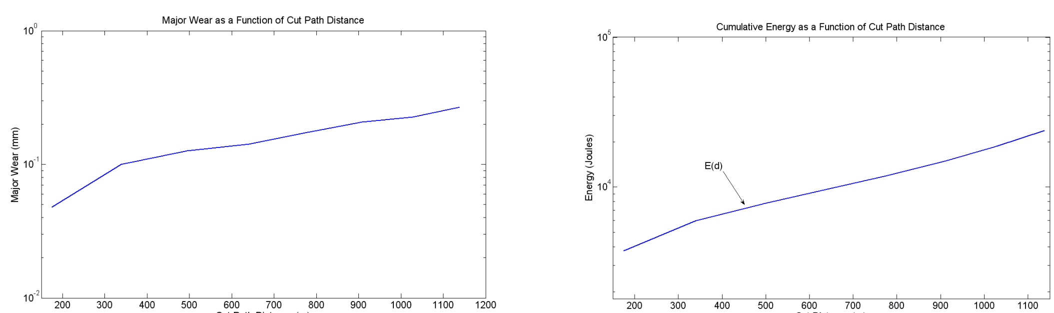

Figure 8. Experimental Major Wear Data (left) and Cumulative Energy Model (right) for Friday

From the results of comparing the major flank wear to the cumulative energy as a function of cut path distance, it is evident that the interaction between the two is scalar. It is particularly interesting that the cumulative energy model is completely based on cut parameters and material properties and provides closely correlated results without curve fitting. For all three tests, the 1st order correlation coefficients between the models and wear data were greater than 0.8, indicating a highly significant scalar relationship.

Discussion of Application

The mechanistic cumulative energy model developed in this work correlates directly to the level of major flank wear on the turning insert tests performed. Although the tests were not performed until catastrophic failure was reached, the major wear data was collected to the point of ISO unacceptable major flank wear of 0.3 mm. The major wear was found to be a scalar reflection of the modeled cumulative cut path energy. Each of the tests was halted at the ISO standard for tool failure is 0.3 mm of major flank wear. Although data from these tests was only taken until 0.3 mm major flank wear, the generation of a tool-workpiece material coefficient, C, was possible. This coefficient was found to have an approximate value of approximately 0.03 for a Tungsten Carbide turning insert cutting AISI 4140 steel. Additionally, the model created theoretical cumulative cut path energy curves based on the initial cutting conditions. Since the cumulative cut path energy is a scalar reflection of the major flank wear, these curves are useful for identifying the wear characteristics of the tool including when the tool should reach an accelerated wear mode towards catastrophic failure. Likewise, the model is also applicable to determining a particular value for major flank wear.

***Application Note*:** For AISI 4140 steel, a Tungsten Carbide insert, and no coolant, the values of C and k are 0.03 and 4e8 respectively. To identify acceptable cut distance, d, evaluate the equation:

$$kn_d = d \cdot T_o \tau_Y W V_x^{-1} \sqrt{V_x^2 + (V_x \tan(90 - Cd \cdot V_x + \alpha_o))^2} \qquad (27)$$

Recalling the variables: T_o =depth of engagement (m), τ_Y =217.5 MPa (AISI 4140), V_x = the surface feed rate (m/s), W =Width engagement, and α_0 =the unworn orientation angle in degrees (Figure 1).

If the equation is evaluated with d as a vector of lengths (from 0 to a chosen value), $[d]$, the major flank wear curve can be plotted as so:

$$n_d = k^{-1}[d] \cdot T_o \tau_Y W V_x^{-1} \sqrt{V_x^2 + (V_x \tan(90 - C[d] \cdot V_x + \alpha_o))^2} \qquad (28)$$

However, the behavior of this equation is continuous with asymptotic inflections. The first vertical asymptote encountered represents the catastrophic failure.

Example: Manufacturing Lab Data

To identify the effectiveness of this model, it must be compared to the trends of actual tool wear data. Although this has already been discussed in the sense that the cumulative cut path energy is closely correlated to the major flank wear, the scalar constant k has not been applied for this correlation. This is because linear correlation is a function of normalized rate of change. However, after the application of the scalar constant, the entire model output can be plotted directly against the experimental wear curve. Figures

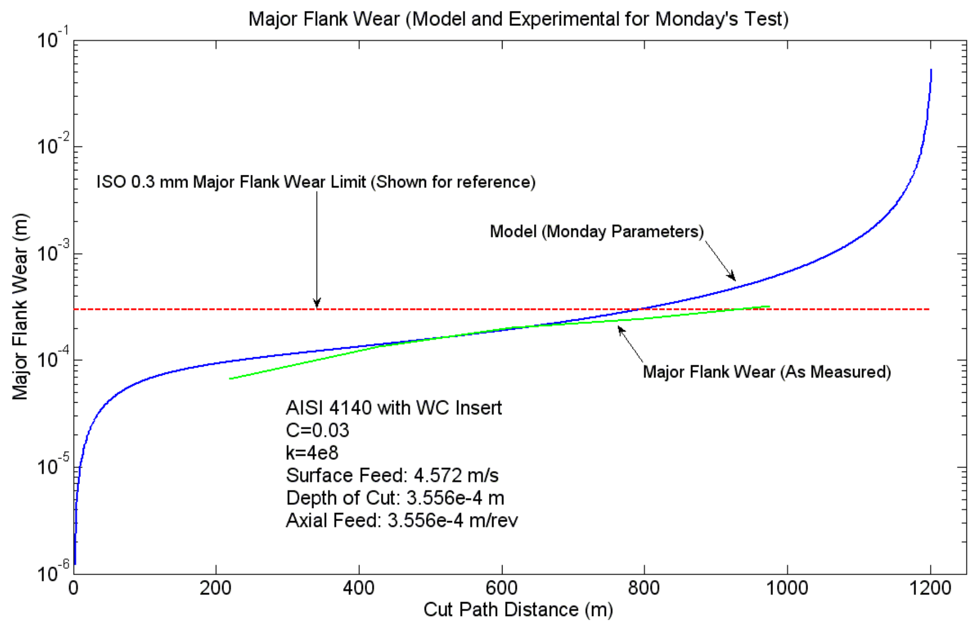

Figure 9. Model and Experimental Major Flank Wear (Monday Lab Test)

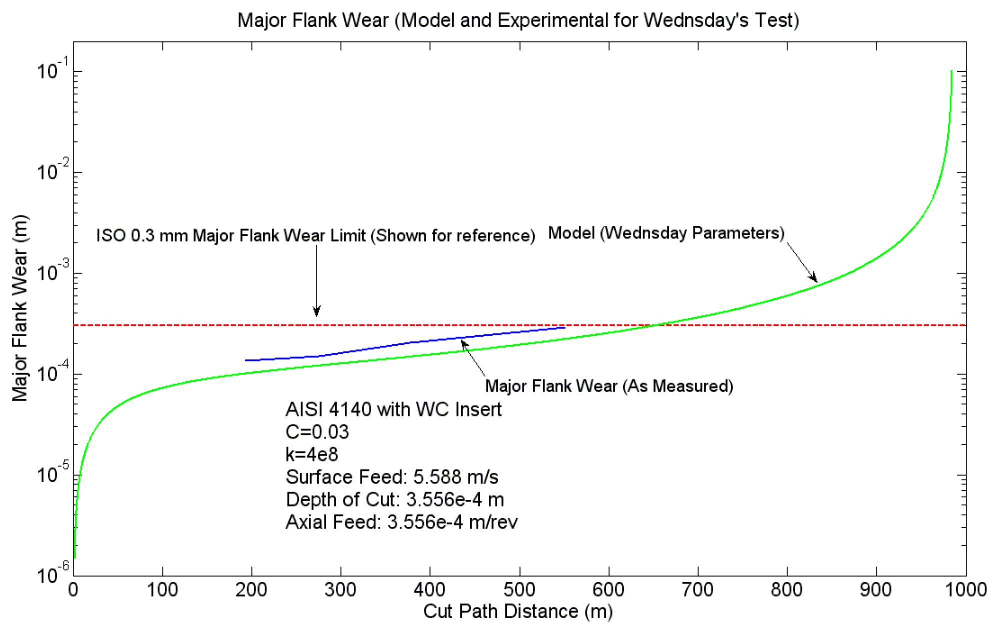

Figure 10. Model and Experimental Major Flank Wear (Wednesday's Lab Test)

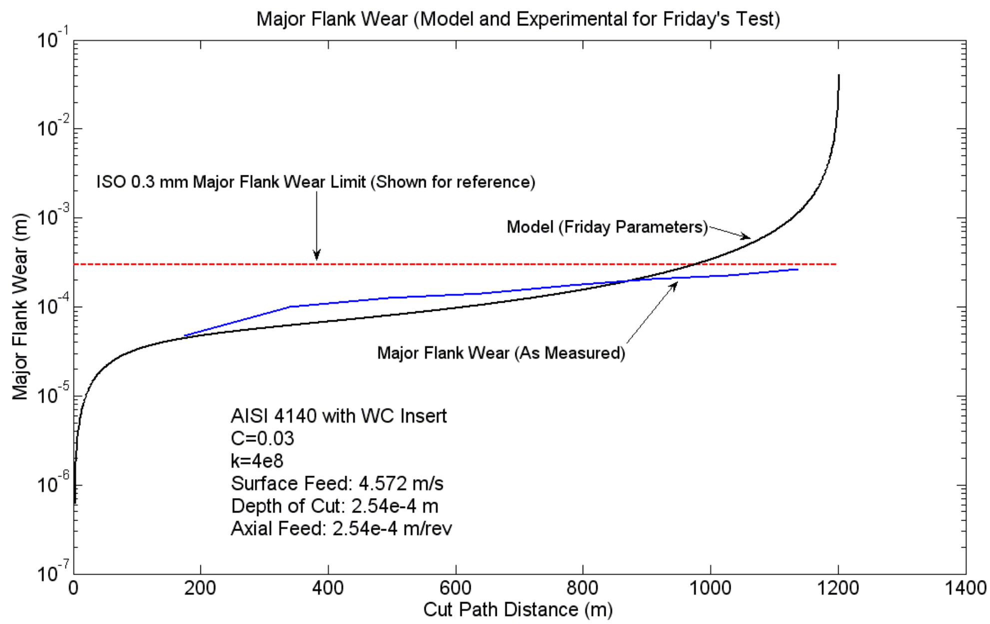

Figure 11. Model and Experimental Major Flank Wear (Friday's Lab Test)

As shown, the theoretical wear-energy model is scalable to the actual wear data through the constant, k. The model shows a good fit to the experimental data over multiple cutting parameters, indicating that the constants C and k are strong functions of material properties (specifically the combination of tool and workpiece materials). This is a desirable outcome and indicates that these constants are not significantly affected by changes in feed or engagements.

Example: Coldfire Untreated Insert Data

Although the model shows an accurate reflection of the major flank wear investigated in the three lab sections, it was desired to have greater resolution on the progression of major flank wear. This was achieved by plotting data acquired during research conducted on the wear behavior of cryogenically treated and untreated tungsten carbide turning inserts. For these comparisons, the untreated tool wear data was used. The tools observed are composed of different carbide quality grades. Additionally, the material being cut is 4340 aerospace grade steel. For this reason, the material property coefficients, C and k, are different. These values were determined to be 0.006 and 4e9 respectively. Like with the three lab tests observed, the cutting parameters are different for several of these tests. The outcome of plotting the model curve against the experimental major flank wear is shown in figures 12 through 20. All of the available untreated Coldfire tests were investigated.

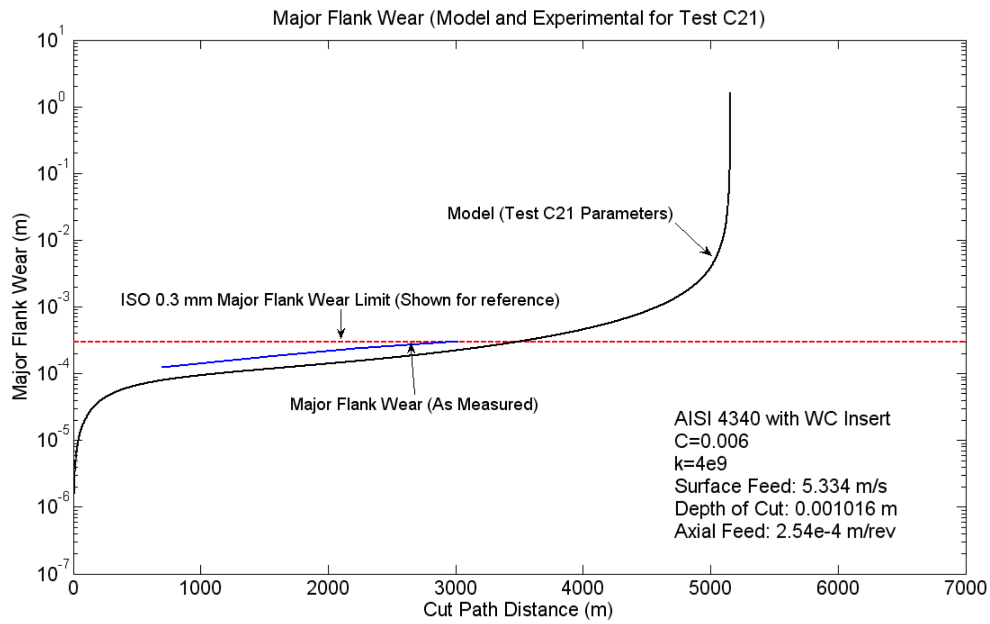

Figure 12. Model and Experimental Major Flank Wear (Untreated Coldfire test C21)

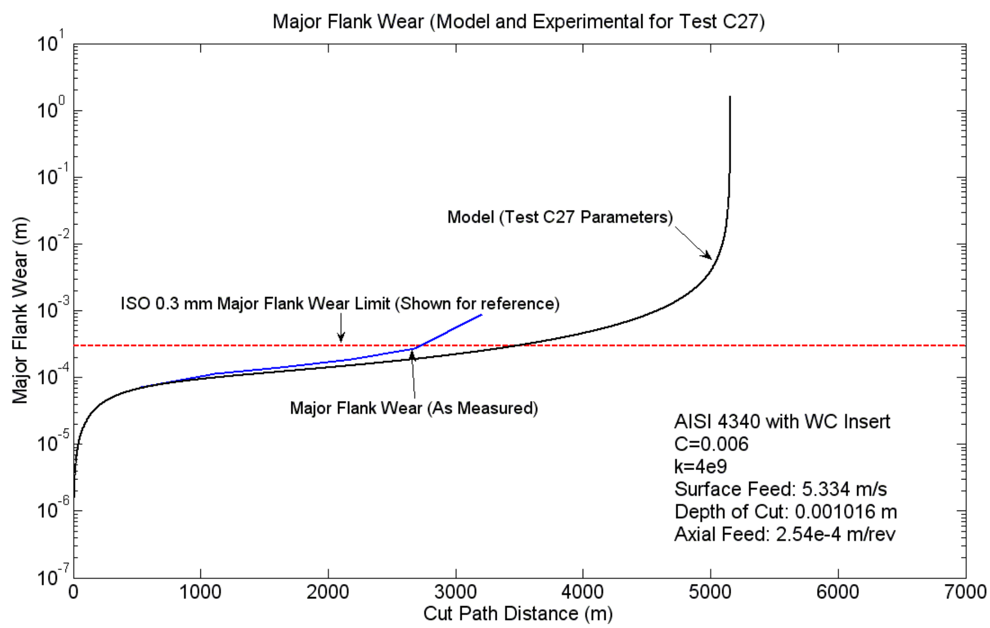

Figure 13. Model and Experimental Major Flank Wear (Untreated Coldfire test C27)

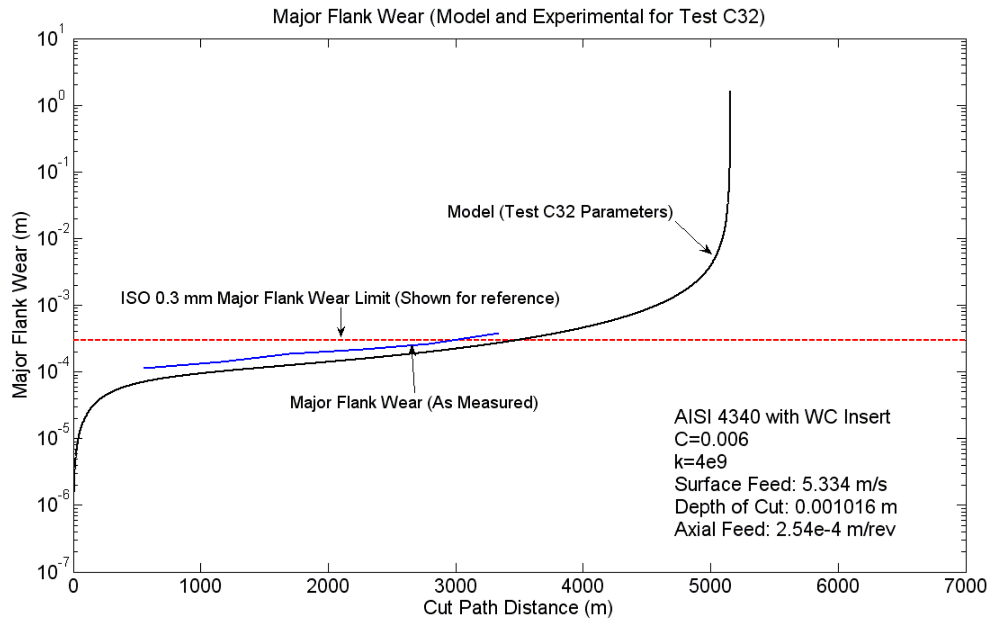

Figure 14. Model and Experimental Major Flank Wear (Untreated Coldfire test C32)

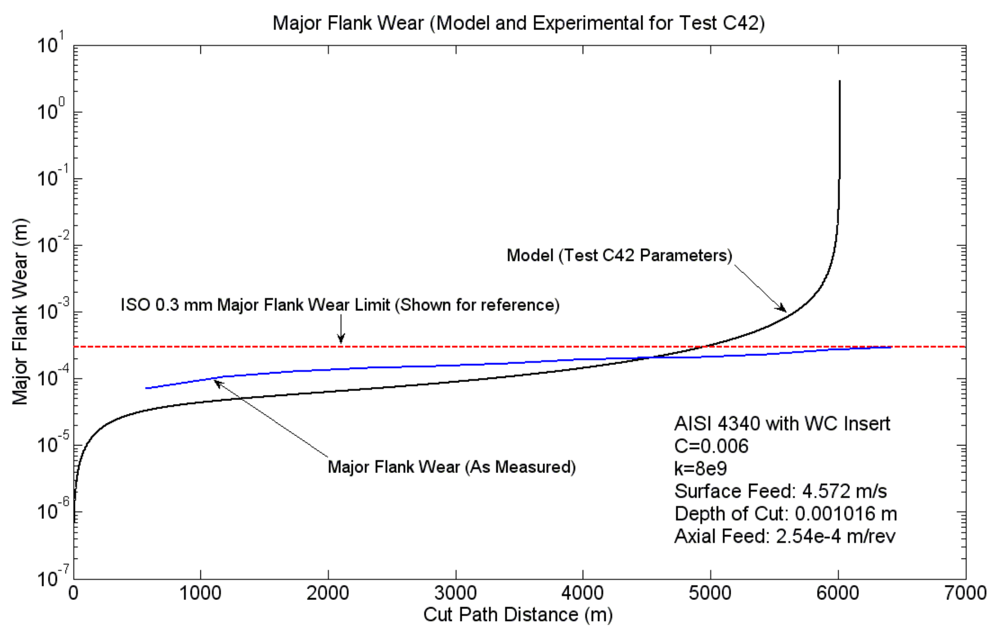

Figure 15. Model and Experimental Major Flank Wear (Untreated Coldfire test C42)

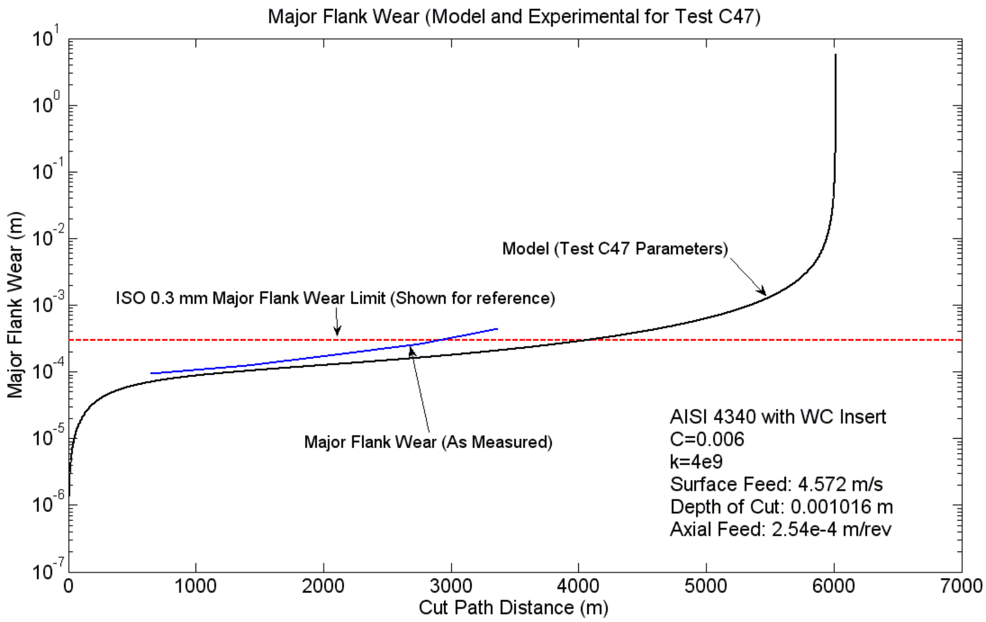

Figure 16. Model and Experimental Major Flank Wear (Untreated Coldfire test C47)

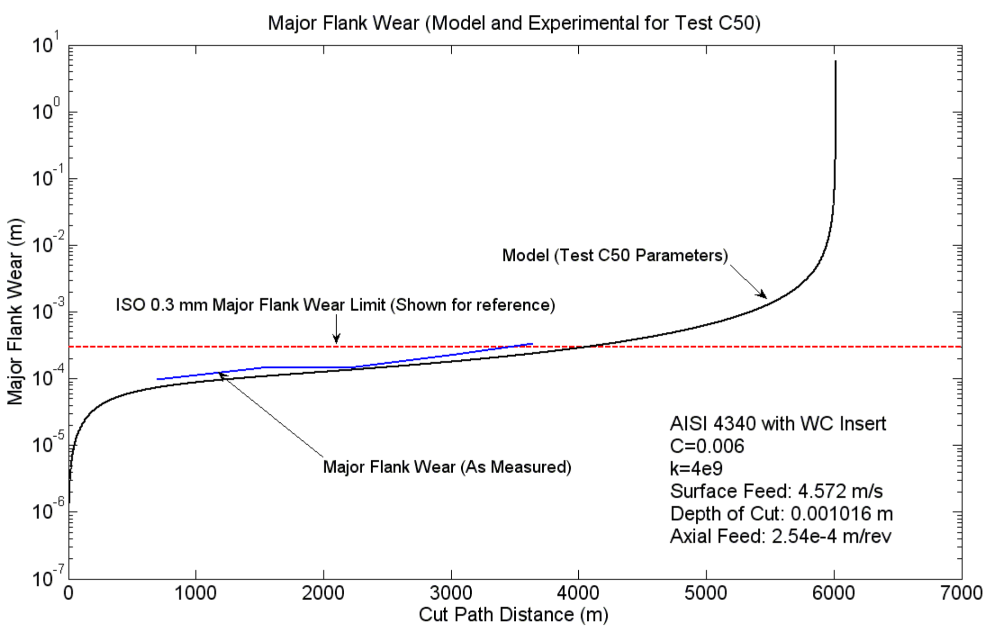

Figure 17. Model and Experimental Major Flank Wear (Untreated Coldfire test C50)

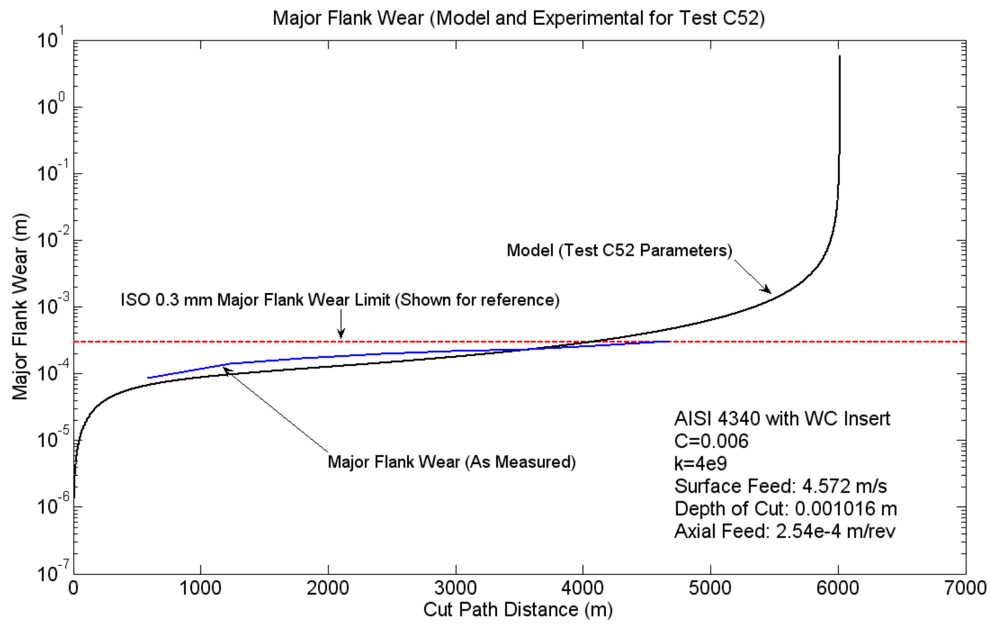

Figure 18. Model and Experimental Major Flank Wear (Untreated Coldfire test C52)

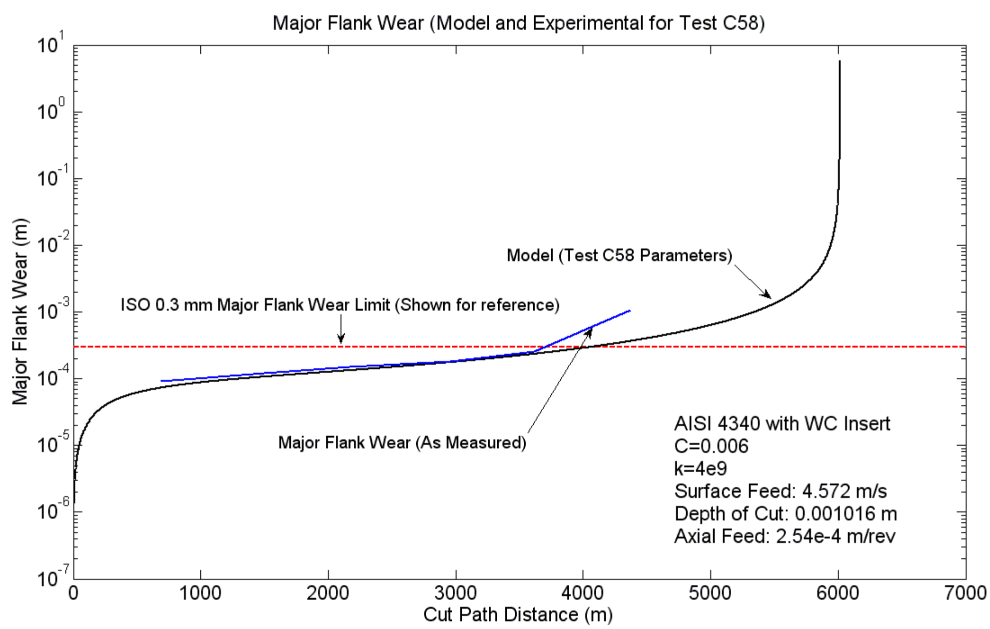

Figure 19. Model and Experimental Major Flank Wear (Untreated Coldfire test C58)

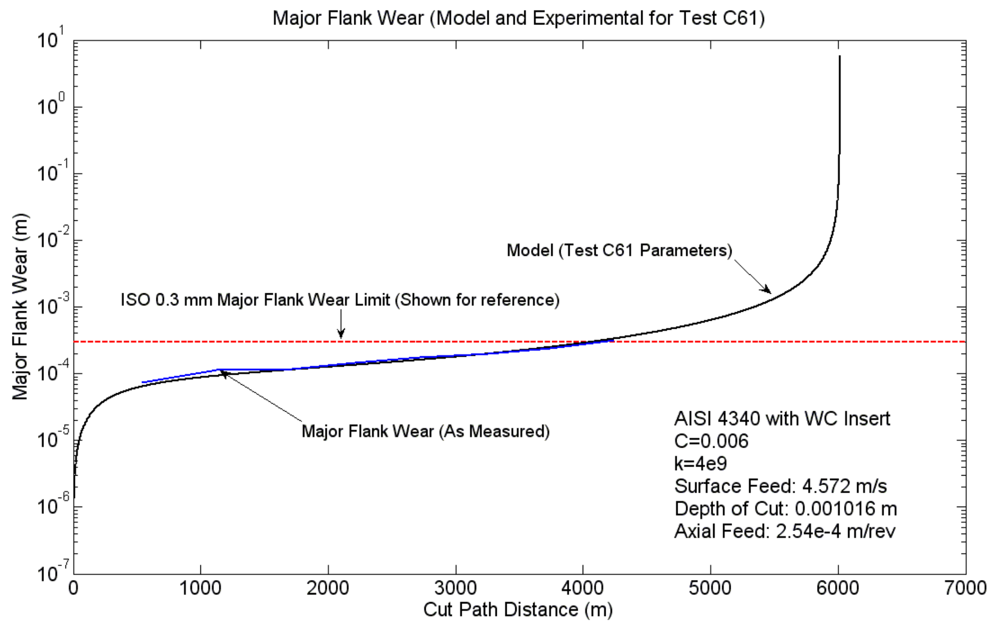

Figure 20. Model and Experimental Major Flank Wear (Untreated Coldfire test C61)

Conclusions from Experimental vs. Model Outcomes

From the data observed, it is seen that the progression of major flank wear is accurately fit by this model. This model is an improvement over exponent curve fits such as the Taylor tool life equation in that it is able to identify the period of tool break-in, linear stable wear, and accelerating wear leading to catastrophic failure. This model is also defined as a function of distance, allowing for an estimation of planned cut path distance by knowing the desired tool wear level. The two constants developed in this model appear to be independent of the cutting parameters, as multiple cutting parameter conditions were investigated over two tool-workpiece material combinations (lab and Colfire). Future work will investigate the relationship between the material properties and the two material property constants developed in this work.